PREDICTION OF FRACTURE OF A FIBROUS COMPOSITE BY THE NUMERICAL EXPERIMENT METHOD

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It is known that it is difficult to obtain a reliable prediction of the fracture conditions for a fibrous composite (FC) on the basis of tests. Thus, for instance, it is impossible to indicate the section of the FC specimen over which a macrocrack will proceed up to the state nearest to fracture [1, 2] from an analysis of acoustic emission (AE) signals. In this connection, electronic computer modeling of the cumulative damage process in an FC acquires special urgency as does the prediction of the site and time of macrocrack development in the specimen from its results.

Two fundamental assumptions [1, 2] underlie modeling of the FC fracture process [3]: the principles of exhaustion of the deformation capacity of the host material under load and the correlation during the cumulative damage process.

Exhaustion of the host deformation capacity can occur with and without crushing of the composite fibers. These two limit cases are apparently the foundation for the introduction of FC fracture classification in [2]: bulk type fracture associated with accumulation of a large number of fiber discontinuities, and dynamical for which the macrocrack is formed because of the discontinuity of one fiber independently of the other fiber discontinuities.

Bulk type FC fracture was assumed during modeling [3], while exhaustion of the host deformation capacity was realized by localization of the plastic deformation in the neighborhood of the site of the fiber discontinuity. As regards the principle of correlation during bulk fracture, then the distinct mutual influence detected in test [1] for the development of the crack formation process in one domain of the specimen on an analogous process in the nearest neighborhood should be emphasized.

The purpose of this investigation is to model the fiber crushing process in a loaded FC specimen on an electronic computer and to predict the dangerous section of the specimen from the data of the numerical experiment.

Analogously to [3] we consider a cylindrical FC specimen of length L bonded by unidirectional fibers of length L and diameter d_f with hexagonal stacking. The spacing between the nearest adjacent fibers is $l = \sqrt{\frac{7}{6\sqrt{3}}}\, d_f \sqrt{\frac{\pi}{v_f}}$ (v_f is the volume fraction of fibers in the FC). The FC specimen is stretched by a constant load σ_0 along the fiber direction at a constant temperature T. The bulk fracture of the FC under the conditions mentioned is determined by the accumulation of thermally activated fiber discontinuities as well as by plastic deformation and splitting of the host.

The tensile stress in the fibers and host equals, respectively

$$\sigma_f = \frac{\sigma_0}{v_f + E_m v_m/E_f}, \quad \sigma_m = \frac{\sigma_0}{v_m + E_f v_f/E_m}$$

($v_m = 1 - v_f$, E_f and E_m are the Young's moduli of the fiber and host).

We assume that the process of fiber crushing is treelike [4] and we examine it under the following assumptions [3].

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1. The mean lifetime of a fiber segment of length x is

$$\tau(x)=\begin{cases}\frac{1}{Cx^p}, & x>l_0, \quad C=\frac{1}{a_0^p\tau_0}\exp\left(-\frac{U_0-\gamma_f\sigma_f}{kT}\right),\\ \infty, & x\leqslant l_0,\end{cases} \tag{1}$$

where U_0, γ_f is the activation energy and the activation volume of the fiber material, k is the Boltzmann constant, $\tau_0 = 10^{-12}$ sec; a_0 is the mean dimension of a structural element of the fiber material, p is a parameter, and ℓ_0 is the effective length of the fiber. The selection of (1) is based on the representation of the "scale" effect for fibers.

2. The probability that a fiber discontinuity will occur at a distance ξ from any of the ends of a segment of length x is

$$\varphi(\xi)\,d\xi=\mu C_{2\mu-1}^{\mu-1}\left(\frac{\xi}{x}\right)^{\mu-1}\left(1-\frac{\xi}{x}\right)^{\mu-1}\frac{d\xi}{x}. \tag{2}$$

The expression (2) describes the correlation between fiber discontinuity sites since it takes account of the drop in the tensile stress at the site of the discontinuity, and the parameter $\mu \geq 1$.

3. Fiber segments have a double numbering. They are characterized by the number i of the fiber and by the number j of the segment of this fiber measured from one of its ends. By using a random number generator realizing a sampling of random lifetimes from an exponential distribution $F(t) = 1 - \exp[-t/\tau(x)]$, an actual lifetime is ascribed to each fiber segment.

Modeling the process of bulk FC fracture starts with the assumption that there are N = 61 entire fibers of length L at the initial time t = 0. The main step in the calculational experiment includes selection of the fiber segment with least actual lifetime, playing the act of its discontinuity in conformity with the distribution (2) on a segment of length ξ and $x - \xi$, calculation of their mean lifetimes $\tau(\xi)$ and $\tau(x - \xi)$ according to (1) and, finally, calculation of their actual lifetimes t_{ij}, T_{ij+1}. Important in the modeling of the process is the treatment of the neighbors of the chosen fiber, where those segments of the nearest fibers that contain a projection of the point of discontinuity of the cracked fiber under the condition of the influence of a crack in the host thereon are considered overstressed. Using the representation [5] we assume that the presence of a stress concentrator at the fiber results in an increase in σ_f thereon. We emphasize that (2) remains valid here for this overstressed segment. In conformity with the described physical model, new lifetimes are calculated for the adjacent segments. Some of them can randomly turn out to be less here than the lifetime of the chosen fiber, which is considered as consistent fiber discontinuities.

The energy

$$\delta E_2 = E(x) - E(x-\xi) - E(\xi),$$

where $E(x)=\frac{\pi d_f^2}{8E_f}\int\limits_0^x \sigma^2(z)\,dz;\quad \sigma(z)=\sigma_f\left[1-\cosh\left(K\frac{z-x/2}{d_f}\right)\Big/\cosh\left(K\frac{x}{d_f}\right)\right];\quad K=8\frac{G_m}{E_f}\frac{v_f^{1/2}}{1-v_f^{1/2}};\ G_m=E_m/(1+\nu_m)$ is liberated for a discontinuity of a fiber of length x at a distance ξ from one of the ends. This energy is expended in the formation of a crack opening in the host and plastic deformation therein near the site of the discontinuity [6].

At present there is no analytic solution of the problem of deformation of a host with a brittle fiber sealed therein during its discontinuity in an exact formulation (see [7], p. 67). However, we use estimates constructed on the basis of the energy conservation law which can approximately be written in our case as

$$\delta E(x) = K(\xi) + K(x-\xi) + \Pi_m + \Pi_f.$$

Here $K(\xi)=\frac{\pi\rho d_f^2}{8}\int\limits_0^\xi \dot U_1^2(\xi')\,d\xi'$, $K(x-\xi)=\frac{\pi\rho d_f^2}{8}\int\limits_\xi^x \dot U_2^2(\xi')\,d\xi'$ are the kinetic energies of segments of the chosen fiber, $\dot{U}_1$ and $\dot{U}_2$ are the segment displacement velocities, $\Pi_m = 2\pi\Gamma_m(r_c^2 - r_f^2)$, $\Pi_f = \frac{\pi}{2}\Gamma_f d_f^2$ are surface energies in the host and fiber, respectively, and Γ_m, Γ_f are the specific surface energies of the host and fiber. There are still no well-founded estimates of the magnitude of the energy δE_1 dissipated in the formation of new surfaces during cracking, and the energy expended in plastic deformation of the host material $\delta E_2 = \delta E - \delta E_1$. Consequently, we introduce the free parameter $\alpha = \delta E_1/\delta E$ to obtain a qualitatively true estimate.

We assume that the crack being formed has the shape of a disc whose radius is $r_c = \sqrt{\delta E_1/(2\pi\Gamma_m)} + d_f^2/4$. We also assume that the nearest fibers "shield" the stress in the host because of the opening of the crack and consequently r_c does not exceed ℓ. We introduce the radius of crack influence r_∞, and we estimate its magnitude by using the method of sections [7]: $r_\infty = (1 + 2/\pi^2) \times r_c$. Fiber overload in our model corresponds to replacement of the parameter C by $C_e = C\exp\left(\beta_i\frac{\gamma_f\sigma_f}{kT}\right)$ in (1) according to the formulas

$$\beta_1 = \frac{1}{6} + \frac{2}{3}\frac{\sigma_m l^2}{\sigma_f d_f^2} \quad \text{for} \quad r_c \geqslant l,$$
$$\beta_2 = \beta_1 - \frac{1}{9\sigma_f d_f^2}\sqrt{\frac{2}{\pi}}K_I(l + 2r_c)(l - r_c)^{1/2} \quad \text{for} \quad r_c < l \leqslant r_\infty, \tag{3}$$
$$\beta_3 = 0 \quad \text{for } r_\infty < l$$

($K_I = 2\sigma_m\sqrt{r_c/\pi}$); formulas (3) are obtained by the method of sections.

The energy residue $\delta E_2 = (1 - \alpha)\delta E$ is expended in plastic deformation of the host near the site of the discontinuity. Analogously to [8], we assume that the governing equation for the host material is

$$\tau_m = \Phi(\gamma_m, \dot{\gamma}_m) \tag{4}$$

(τ_m, γ_m are the shear stresses and strains in the host that occur during motion of the ends of the chosen fiber at the velocity $\dot{U}$). It is shown in [8] that if the velocity of boundary displacement of the half-space of the material (4) exceeds $|\dot{U}^*|$ that the solution of simple plastic wave type yields localization of the deformation γ_m in the near-boundary layer of thickness δ called wave trapping. It is assumed here that for a certain magnitude of the plastic deformation γ_m^* the derivative is $d\tau_m/d\gamma_m = 0$, and $\dot{U}^*$ is related to γ_m^* by the formula

$\dot{U}^* = -\int_0^{\gamma_m^*} c(\gamma_m)\,d\gamma_m$ [$c(\gamma)$ is the speed of sound in the host as a function of the deformation

$c(\gamma_m) = \sqrt{\frac{d\tau_m}{d\gamma_m}/\rho_m}$, and ρ_m is the host density].

The governing equation

$$\tau_m/\tau_{0m} = f(\gamma_m/\gamma_{m0})[1 + p_1\ln(1 + \dot{\gamma}_m/\dot{\gamma}_{m0})] \tag{5}$$

is selected in [8] ($f(z) = z^{n_1}/(1 + az^{n_1+1})$, τ_{0m}, γ_{m0}, a, n_1, p_1, $\dot{\gamma}_{m0}$ are constants of the host material). Numerical computations [8] showed that for $n_1 = 0.02$ and $a = 0.005$ for (5) the thickness δ of the near-boundary layer containing strongly deformed material can be estimated as

$$\delta = 0.014c_0^2/\dot{\gamma}_{0m}(\dot{U} + \dot{U}^*) \tag{6}$$

for $0 < |\dot{U}| < c_0$, where $c_0 = \sqrt{\tau_{0m}/\rho_m}$; $p_1 = 0.05$; $\gamma_{0m} = 0.01$. Data are presented in [8] about the constants for weak steel: $c_0 = 200$ m/sec, $\dot{U}^* = 30$ m/sec, $\dot{\gamma}_{m0} = 10^5$ sec^{-1}. We obtain $\delta = 40$ µm for the mentioned magnitudes of the parameters and $\dot{U} = 100$ m/sec. We assume that (6) can be an estimate of the size of the domain of significant plastic deformations. It is reasonable to assume that the deformation capacity of the host is exhausted in these domains, which results in a diminution of Γ_m therein and an increase in r_c and r_∞, and, taking (3) into account, to overload of the adjacent fibers. For simplicity in the computations we consider $\dot{U} \sim (\delta E_2)^{1/2}$.

In this model the FC specimen is divided into m layers perpendicular to the fiber direction. For a fiber discontinuity in the k-th layer, a crack and a domain of large plastic deformations in the form of a cylinder with axis on a given fiber of diameter 2δ and height L/m are formed. If a crack is formed in the adjacent fiber, whose path passes through a domain of large deformations, then its radius is calculated by starting from the assumption of an effective specific surface energy Γ_e. If the crack lies completely in the domain of strongly deformed material $\Gamma_e = \Gamma_m/s$. If the crack passes partially through the domain of strongly deformed material, then Γ_e is calculated as the mean value of Γ_m and Γ_m/s in proportion to the areas trapped by the crack in the strongly F_1 and weakly F_2 deformed host domains $\Gamma_e = \left(\Gamma_m F_2 + \frac{\Gamma_m}{s} F_1\right) / (F_1 + F_2)$.

The following values of the constants were chosen for the computations: $E_f = 4 \cdot 10^4$ kgf/mm², $d_f = 0.14$ mm, $v_f = 0.5$, $\gamma_f = 300$ A³ (boron fibers), $\Gamma_m = 1$ kJ/m², $E_m = 7200$ kgf/mm² (aluminum host), L = 50 mm, $\sigma_0 = 20$ kgf/mm², $\ell_0 = 3$ mm, the parameter p = 1.5 in (1), α = 0.5, s = 5, and the parameters $\mu = 2$ in (2). Since the set of constants from (5) for the chosen host is unknown to the authors, constants corresponding to steel [8] are taken for a qualitative estimate.

The time dependence is shown in Fig. 1: *a* is the rate of accumulation of single discontinuities of fibers (curve 1), consistent doubles (2), triples (3), and discontinuities of four and higher multiplicity (4), b is the power liberated for fiber discontinuities of appropriate multiplicity. It is seen that single discontinuities predominate in the early stage of the process of fiber discontinuity accumulation, they yield the main contribution to the liberated power W. The sequence of "inclusions" of the mechanisms of consistent discontinuities of growing multiplicity as well as the trapping action of the mechanisms as the fiber discontinuity sites are exhausted is seen clearly. Special attention should be turned to the noticeable growth of the number of multiple discontinuities, substantially exceeding the quantity of single fiber discontinuities. According to [1, 2], the appearance of "consistent" discontinuities is the predecessor of specimen macrofracture.

Let us now consider a more exact method of predicting the site for the occurrence of a mainline crack by starting from the assumption [1, 2] that macrofracture sets in in that section where energy liberation per unit time is greater. Under such an assumption it is important to calculate the probability of deviation of the magnitude of elastic energy being liberated for fiber discontinuities per unit time in a given specimen section from its mean value ("burst" of power).

Let us introduce the random vector $\mathbf{W}_k(t)$ whose components are random values of the power being liberated in a given section to a given time t. Following the ideas of the theory of probability of large variances [9], we introduce a sequence of n identical FC specimens and assume that $\mathbf{W}_1(t), \mathbf{W}_2(t), \ldots, \mathbf{W}_n(t)$ is a sequence of independent identically distributed random vectors of the power being liberated in this specimen. We consider the random vector distribution

$$\frac{\mathbf{W}_1(t) + \mathbf{W}_2(t) + \ldots + \mathbf{W}_n(t) - nM\mathbf{W}_k(t)}{B_n}$$

(B_n is a sequence that tends to infinity more rapidly than $n^{1/2}$ but more slowly than n). Let P(...) denote the probability of the event indicated in the parentheses, then according to [9], the following relationship is proved

$$\lim_{\delta \to 0} \lim_{n \to \infty} \frac{n}{B_n^2} \ln P \left\{ \left| \frac{\mathbf{W}_1(t) + \ldots + \mathbf{W}_n(t) - nM\mathbf{W}_k(t)}{B_n} - x \right| < \delta \right\} = -\frac{1}{2} \sum_{ij} a_{ij} x^i x^j, \qquad (7)$$

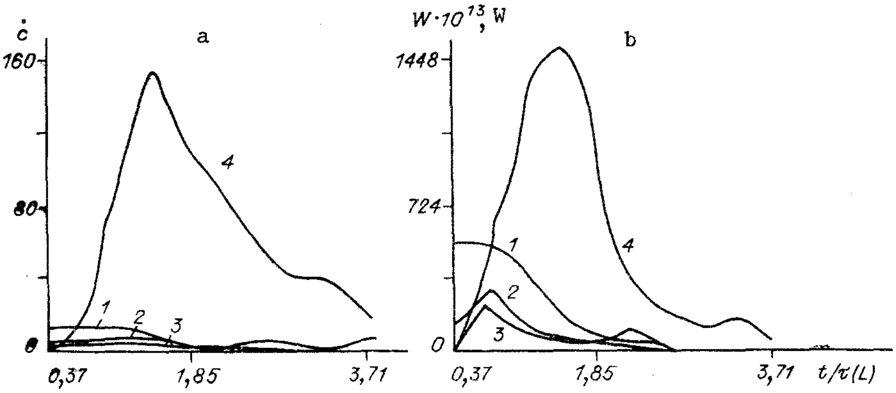

Fig. 1

i.e., the distribution of normalized deviation of the sum of random vectors $\mathbf{W}_k(t)$ from its mean value is a normal distribution given by the matrices a_{ij}, where a_{ij} is the inverse matrix of covariation for the random vectors mentioned, and $MW_k(t)$ is the mathematical expectation of the vector $\mathbf{W}_k(t)$.

Let us determine the covariation matrix A_{ij} as an empirical covariation matrix [10]. An electronic computer calculation of the matrix $a_{ij} = A_{ij}^{-1}$ of dimensionality $m \times m$ is difficult for large dimensionalities $m \geq 10$ because of the incorrectness of its formation, consequently, we limit ourselves in the computations to selecting eight sections of the FC specimen. The maximal time of the cumulative damage process t_m is given in the initial data and calculation of the matrix A_{ij} is performed at the times $t_i = t_m/10i$ $(i = 1, \ldots, 10)$. It should be recalled that (7) has been obtained to the accuracy of logarithmic equivalence [9]; consequently, only comparative estimates of the fracture probability are possible here for sections of an FC specimen. Calculations displayed the positive-definiteness of the matrix a_{ij} at the time t_i, then the derivative of the right side of (7) with respect to the direction will characterize the degree of its growth. In our case it is important to determine the direction of the maximal change (7), i.e., the number of the FC specimen section for which the greatest probability of power "bursts" should be predicted. The direction cosines n_i determine this direction. In particular for $x^i = x$ in (7)

$$n_i = \frac{\sum\limits_{k=1}^{m} a_{ik}}{\sqrt{\sum\limits_{i=1}^{m}\left(\sum\limits_{k=1}^{m} a_{ik}\right)^2}}, \tag{8}$$

and the absolute value of the derivative with respect to the given directions is

$$q = |\operatorname{grad}\left(\sum a_{ij}x^i x^j\right) = \sqrt{\sum_{i=1}^{m}\left(\sum_{k=1}^{m} a_{ik}\right)^2}\, x. \tag{9}$$

Utilization of (8) and (9) to predict the probability of power "bursts" in this section is substantially a linear extrapolation on the basis of information accumulated in the matrix a_{ij} up to a given time. Computations showed that n_i take on negative values at certain times. Hence, growth of the probability of power "bursts" in an appropriate FC specimen section follows.

We present two examples of predicting the site of macrocrack appearance, associated with power "bursts" during fiber discontinuity. It is interesting to compare the results of predictions in the case of taking account of the correlation between discontinuities in adjacent fibers (the parameter $s = 5$), along an individual fiber (the parameter $\mu = 2$) and in the absence of correlation ($s = 1$, $\mu = 1$). The magnitude of the correlation of fiber discontinuities depends on the FC structure, it governs the tempo of cumulative damage and the localization.

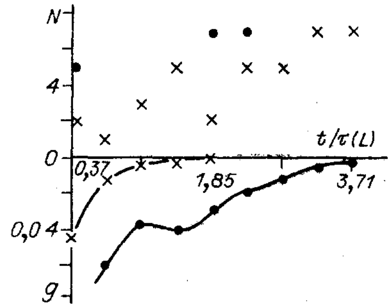

Fig. 2

The diagram for predicting the site of a dangerous FC specimen section as a function of the dimensionless time $t/\tau(L)$ is shown in Fig. 2 for the first computation (crosses) and for the second (dots). The ordinates of the points in the upper part of the diagram indicate the location of the dangerous FC specimen section while the corresponding ordinates of points in the lower part characterize the change in probability of power "bursts" being liberated in the specimen ($n = 25$ is taken in the computations).

Let us now discuss the results obtained from the viewpoint of correlation of the fiber discontinuities. Computations showed that for selected values of the parameters the intensity of fiber crushing is higher in the first case than in the second. Thus, for instance, about 1000 fiber discontinuities occur in the first computation in an identical time interval $t_m = 3.71\tau(L)$ and 180 in the second. Multiple "consistent" discontinuities are almost absent in the second, which would indeed result in a small number of fiber discontinuities, and also individual FC sections are almost not extracted from the viewpoint of the possibility of a large "burst" of liberated elastic energy. At the same time, as follows from Fig. 2, dangerous sections appeared in the first computations from the very beginning of the cumulative damage process, which is apparently a reflection of the presence of "consistent" fiber discontinuities.

In conclusion, let us note that the proposed method for predicting FC fracture can be used to process experimental data obtained from an analysis of AE signals, say, during loading of real FC specimens since the AE signal amplitude is proportional to the magnitude of the fiber elastic energy being liberated during their discontinuity. The location of AE signals permits distinguishing signals emitted from different sections of the specimen [1, 2] and computation of the empirical covariation matrix in values of random power vectors according to AE signals is the basis for estimating the prediction of a dangerous FC specimen section.

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EFFECTIVE MODULI OF MULTIPHASE MATRIX COMPOSITES

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An extensive literature [1-3] addresses the question of calculating the effective characteristics of granulated matrix composites. However, these studies are generally concerned with two-phase composites, i.e., composites consisting of a matrix with inclusions having the same physical and geometric characteristics. The polydisperse model proposed by Hashin [4] has several important deficiencies which make it unsuitable for the design of actual composites: first of all, it is invalid for multiphase mixtures whose fractions differ in density; second, it does not account for the geometry of the filler and the associated arrangement of the filler material in the matrix.

In the present study, we construct a theory to calculate the effective moduli of particulate matrix composites which is free of these problems. Our theory is in turn based on the theory of composite media proposed by Hill [5] and a generalized singular approximation of Shermergor's theory of random functions [1]. As an example of the use of the results obtained here, we examine the determination of the elastic moduli of polymer composites consisting of a polymer matrix and whole spherical inclusions introduced into the matrix.

We will study a medium consisting of a homogeneous, isotropic matrix and spherical or ellipsoidal particles introduced into the matrix. The introduced particles are randomly located and oriented in the matrix. It is assumed that the filler consists of n − 1 isotropic phases differing in density and elastic characteristics and − in the case of spheres − in external diameter. Given the volume content of inclusions in the composite v_s, it is assumed that we know the histogram describing the distribution of the phases with respect to their volume content in the filler $v_s^{(i)}$. The latter quantity is determined by the vector function

$$\mathbf{p} = \mathbf{p}(p_1, p_2, \ldots, p_{n-1}); \quad \sum_{i=1}^{n-1} p_i = 1 \tag{1}$$

so that $v_s^{(i)} = p_i v_s$.

If the components of the filler differ in density, then the below vector-function describing the distribution of the densities of the phases is assigned

$$\boldsymbol{\rho}_s = \boldsymbol{\rho}_s(\rho_s^{(1)}, \rho_s^{(2)}, \ldots, \rho_s^{(n-1)}). \tag{2}$$

In the case when the filler is spherical, we should also know the histogram describing the distribution of the fractions with respect to external diameter

$$\mathbf{d} = \mathbf{d}(d_1, d_2, \ldots, d_{n-1}). \tag{3}$$

Equations (1)-(3) describe the structure and geometry of the filler of a particulate matrix composite.

In accordance with the generalized singular approximation [1], the tensor of the effective moduli of a multiphase nonmatrix mixture is found from one of the equivalent expressions

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